such an approximation is valued whenever phase and group velocities are not too different from each other. It is the purpose of the present letter to show that this may not be a correct criterion to choose. In fact, even though group and phase velocities differ by a small amount the above approximation may still produce an imperfect temperature compensation (i.e., $\alpha_\phi \neq 0$). In the following, a quantitative evaluation of this phenomenon is presented in conjunction with "unapproximated" design formulas for the TLDL.

Let us use the primed quantities $l'_{A,B}$ to indicate the solutions to the system of (1) and (3). They are found to be [1]:

$$l'_A = \frac{\alpha_B \tau C}{(\alpha_B + |\alpha_A|)(1+\Delta_A)\sqrt{\epsilon_{\text{eff}\,A}}}$$

$$l'_B = \frac{|\alpha_A| \tau C}{(\alpha_B + |\alpha_A|)(1+\Delta_B)\sqrt{\epsilon_{\text{eff}\,B}}} \quad (4)$$

where C is the velocity of light in vacuum, $\Delta_{A,B} = (\omega/2\epsilon_{\text{eff}\,A,B})(\partial \epsilon_{\text{eff}\,A,B}/\partial\omega)$, $\epsilon_{\text{eff}\,A,B}$ are the effective dielectric constants of delay lines (A) and (B), and ω is the angular operation frequency. Note that $1+\Delta$ is the ratio between phase and group velocity along a transmission line.

When $l'_{A,B}$ are substituted into (2), it turns out that

$$\left(\frac{l'_A}{v_{\text{ph}\,A}} + \frac{l'_B}{v_{\text{ph}\,B}}\right)^{-1}\left(\frac{-|\alpha_A| l'_A}{v_{\text{ph}\,A}} + \frac{\alpha_B l'_B}{v_{\text{ph}\,B}}\right) = \alpha_\phi^* \neq 0. \quad (5)$$

The fact that $\alpha_\phi^* \neq 0$ indicates that the approximate lengths $l'_{A,B}$ cause the transmission phase of the device to be thermally unstable. From (5), via (3) and (4) α_ϕ^* may be cast under the form

$$\alpha_\phi^* = \frac{\dfrac{1+\Delta_A}{1+\Delta_B} - 1}{\dfrac{1}{\alpha_B}\dfrac{1+\Delta_A}{1+\Delta_B} + \dfrac{1}{|\alpha_A|}}. \quad (6)$$

From (6) it is recognized that, given two substrate materials with α_A and α_B, it is sufficient that $1+\Delta_A = 1+\Delta_B$ to have $\alpha_\phi^* = 0$. However this condition is unrealistic as, in general, the two MIC's have different frequency dispersion. In a practical situation with $1+\Delta_A \neq 1+\Delta_B$, α_ϕ^* is different from zero and may be calculated by use of (6).

For the case reported in [1] of a composite TLDL with $BaTi_4O_9$ and Al_2O_3 substrates, $|\alpha_A| = 3.9 \times 10^{-6}/°C$, $\alpha_B = 80.1 \times 10^{-6}/°C$, $\Delta_A = 1.077$, $\Delta_B = 1.033$, and $\alpha_\phi^* = 0.148 \times 10^{-6}/°C$. Whether this value of α_ϕ^* is acceptable depends on the system's specs. Note that the measured value of the total transmission phase temperature coefficient α_ϕ reported in [1] is $0.6 \pm 0.3 \times 10^{-6}/°C$.

A different situation wherein the approximation is certainly not valid because α_ϕ^* is a considerable portion of the maximum acceptable α_ϕ, however, may be encountered in practice for materials with different physical properties than those reported in [1]. In fact, one may wish to use MIC's with higher negative dielectric constant temperature coefficients and compensate them with MIC's on substrates other than single crystal sapphire (e.g., with ceramic allumina). For instance a type of commercially available $BaTi_4O_9$ exists with $|\alpha_A| = 15 \times 10^{-6}/°C$ [2]. Using this material together with allumina ($\alpha_B = 50 \times 10^{-6}/°C$) we built a composite TLDL operating at 14.125 GHz with a group delay time of 16.66 ns, corresponding to a 1-symbol duration in a 120 Mbit/s DC-QPSK signal. In this device the characteristic impedances of the two partial delay lines at zero frequency were 30 Ω and 50 Ω, respectively.

As a consequence $1+\Delta_A/1+\Delta_B = 1.0471$ and $\alpha_\phi^* = 0.538 \times 10^{-6}/°C$. This value of is 34.3 percent of the spec value of $\alpha_\phi = 1.57 \times 10^{-6}/°C$ corresponding to a stability of ± 2 degrees over a temperature interval of 30°C. Under these circumstances the approximation of [1] is not accurate and the partial delay line lengths l_A and l_B must be calculated using the following "exact" formulas obtained from the (1) and (2)

$$l_A = \frac{\alpha_B \tau C}{[\alpha_B(1+\Delta_A) + |\alpha_A|(1+\Delta_B)]\sqrt{\epsilon_{\text{eff}\,A}}}$$

$$l_B = \frac{|\alpha_A| \tau C}{[\alpha_B(1+\Delta_A) + |\alpha_A|(1+\Delta_B)]\sqrt{\epsilon_{\text{eff}\,B}}}. \quad (7)$$

On the basis of the above results we conclude that, in general, formulas (4) are a good approximation to formulas (7) whenever the substrate material with negative dielectric constant temperature coefficient is highly stable, i.e., α_A is very small. Furthermore, inspection of (6) reveals that this result is pretty much independent of the value of α_B for all practical situations wherein $1+\Delta_A/1+\Delta_B \simeq 1$.

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Periodically Loaded Transmission Lines

JOSE PERINI

***Abstract*—In this paper equations for the transmission parameters of a periodically loaded line are derived in closed form with no restriction on the size, type, or number of discontinuities. The equations also take into consideration any attenuation that may exist on the line.**

Several plots of the input reflection coefficient and transmission coefficient are presented and compared with experimental results. The agreement is very good.

I. Introduction

It is common practice to try to maximize the transfer of power to a load at the end of a long transmission line by minimizing the input reflection coefficient of the system. In this paper it is shown that if the line has periodically distributed discontinuities, which is quite common, then this procedure may lead to quite the opposite result. More recently, interest in the switching characteristics of pulses in such lines has resulted in their being approximately analyzed. [1]. Pulse switching is an important problem in the design of high-speed digital computers. In this paper a closed form solution for the periodically loaded line is obtained. The equations can handle any type of discontinuity

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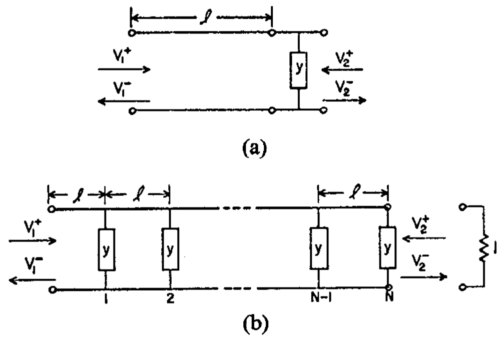

Fig. 1. (a) Transmission line cell. (b) Periodically loaded line.

and attenuation on the line. The only restriction is that the discontinuities must be identical and periodically distributed. By appropriate limiting procedures many of the results reported elsewhere can be easily derived [2]–[5]. Comparison with experiment shows very good agreement.

II. Theoretical Treatment

Consider the transmission parameters [6] of a cell of the transmission line as shown in Fig. 1(a). It consists of a length of line l of characteristic admittance Y_0 and a shunt admittance y (normalized with respect to Y_0). The transmission matrix is

$$T=\frac{1}{1+\Gamma}\begin{bmatrix}(1+2\Gamma)e^{-\gamma l} & \Gamma e^{\gamma l}\\ -\Gamma e^{-\gamma l} & e^{\gamma l}\end{bmatrix} \tag{1}$$

where $\gamma=\alpha+j\beta$ is the propagation constant, α being the attenuation per meter and $\beta=2\pi/\lambda$. In (1) Γ is the reflection coefficient of y in parallel with the characteristic admittance of the line.

If N identical sections are cascaded as shown in Fig. 1(b), then the total transmission matrix will be

$$T_N=T^N. \tag{2}$$

To raise T to the Nth power the Chebyschev polynomials of second kind

$$U_{N-1}(z)=\frac{\sin h[N\cos h^{-1}z]}{\sin h[\cos h^{-1}z]} \tag{3}$$

can be used as suggested in the literature [7], [8]. Therefore,

$$T_N=TU_{N-1}(z)-IU_{N-2}(z) \tag{4}$$

where I is the identity matrix and z is given by

$$z=\frac{1}{2(1+\Gamma)}\left[e^{\gamma l}+(1+2\Gamma)e^{-\gamma l}\right]. \tag{5}$$

Equations (4) and (5) solve the problem completely.

Some of the quantities of interest are the input reflection coefficient and the transmission coefficient of the whole line when terminated by the characteristic admittance. This is readily obtained from (4). The input reflection coefficient is

$$\Gamma_N=\left.\frac{V_1^-}{V_1^+}\right|_{v_2^+=0}=\frac{\Gamma e^{-2\gamma l}U_{N-1}(z)}{U_{N-1}(z)-(1+\Gamma)e^{-\gamma l}U_{N-2}(z)} \tag{6}$$

and the transmission coefficient is

$$\tau_N=\left.\frac{V_2^-}{V_1^+}\right|_{v_2^+=0}=\frac{(1+\Gamma)e^{-\gamma l}}{U_{N-1}(z)-(1+\Gamma)e^{-\gamma l}U_{N-2}(z)}. \tag{7}$$

III. Important Results

From (6) and (7) some interesting and important results can easily be derived [9]. Note that they furnish magnitude and phase information of Γ_N and τ_N. A list of some of the more important results follows.

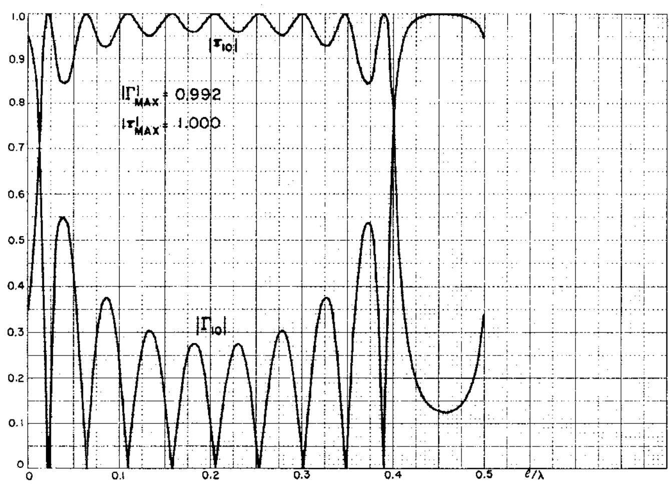

Fig. 2. Capacitive discontinuities. $|\Gamma|=0.27$. $N=10$.

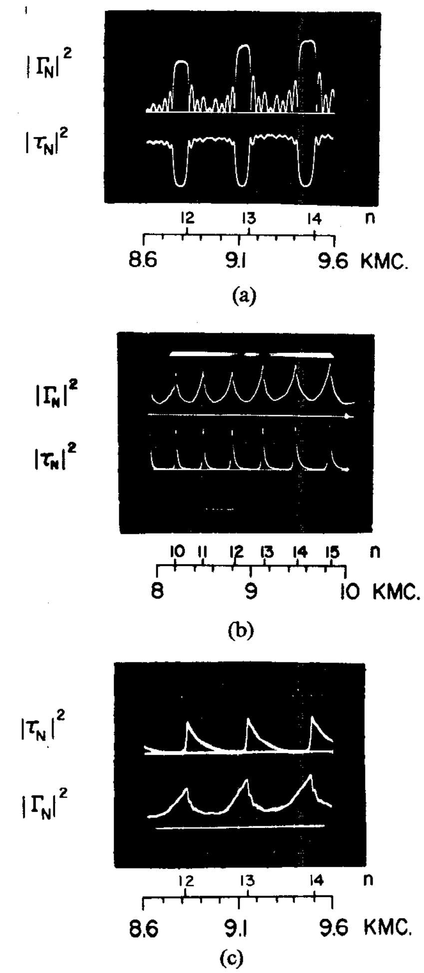

Fig. 3. (a) 10 capacitive discontinuities, $|\Gamma|=0.27$. (b) 10 resistive discontinuities, $|\Gamma|=0.33$. (c) 10 lossy capacitive discontinuities. $\delta\simeq 45°$, $|\Gamma|=0.39$.

1) For reactive discontinuities the maximum Γ_N and the minimum τ_N are coincident (and vice versa). From energy considerations this should be expected (lossless lines). See Fig. 2 and 3.

2) For resistive discontinuities the maximum τ_N corresponds to the maximum Γ_N. This was somewhat surprising. See Figs. 4 and 3(b).

3) For reactive discontinuities, lossless lines and $m\frac{\lambda}{2}\leqslant l<(m+1)\frac{\lambda}{2}$, $m=0,1,2,\cdots$, there will be $N-1$ values for l for which $\Gamma_N=0$.

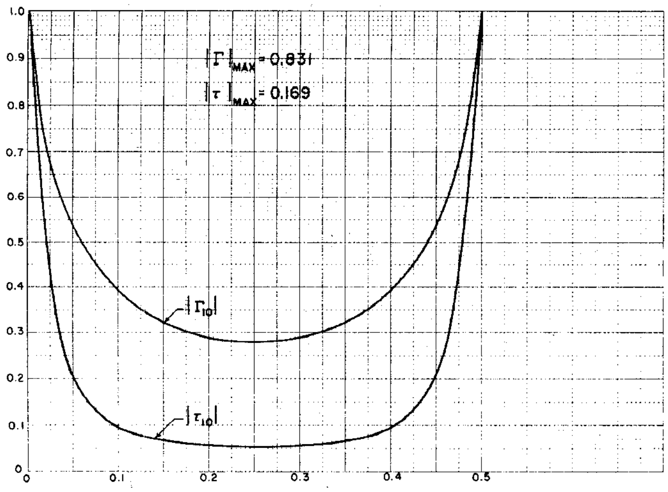

Fig. 4. Resistive discontinuities. $|\Gamma| = 0.33$. $N = 10$.

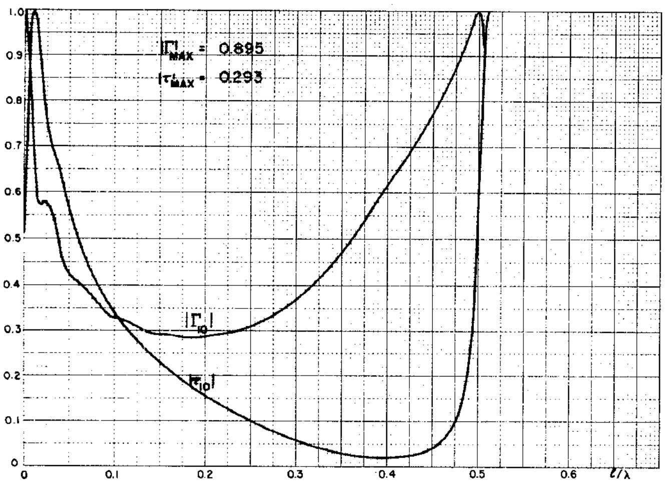

Fig. 5. Lossy capacitive discontinuities. ($\tan\delta \simeq 1$). $|\Gamma| = 0.39$. $N = 10$.

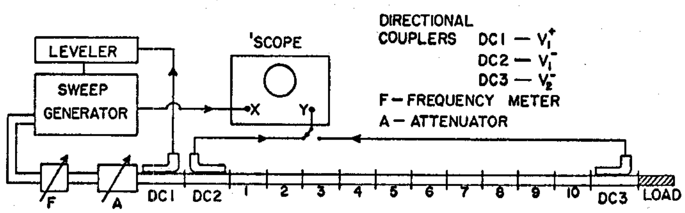

Fig. 6. Experimental setup.

4) For discontinuities which are a general admittance there is no simple relation between the maxima and the minima of Γ_N and τ_N. See Figs. 5 and 3(c). τ_N exhibits a minima that should be avoided if maximum power transfer is sought.

5) For resistive discontinuities the maximum Γ_N and τ_N will occur at multiples of $\lambda/2$ and the minimum at odd multiples of $\lambda/4$.

6) For the case of discontinuities which are not purely resistive the maximum of Γ_N will occur when l is slightly shorter or slightly larger than an even multiple of $\lambda/2$. These cases will correspond to inductive or capacitive discontinuities, respectively.

7) In the case of discontinuities which are a general admittance τ_n and Γ_N will have an assymmetrical behavior as shown in Figs. 5 and 3(c). This is important when maximum transfer of power is sought.

8) For lossless lines Γ_N and $|\tau_N|$ will be periodic functions of l with period $\lambda/2$.

9) For small discontinuities the input reflection coefficient behaves like the array factor of N equally spaced antennas (5).

IV. Experimental Verification

In order to verify some of the results obtained by (6) and (7) the setup of Fig. 6 was used with 10 sections of waveguide. The discontinuities were introduced at each joint. Instead of varying the distance between the discontinuities the frequency was swept in an appropriate range. As the discontinuities used change very slowly with frequency the results are essentially the same as varying the length of the waveguide sections. The agreement with the computed results is very good as seen in Fig. 3(a), (b), and (c) and Figs. 2, 4, and 5.

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Resonant Frequencies of Rectangular Dielectric Resonators

J. F. LEGIER, P. KENNIS, S. TOUTAIN, AND J. CITERNE

***Abstract*—The resonant frequencies of isolated dielectric resonators of rectangular shape are calculated using the dielectric waveguide model. The waveguide treatment of the rectangular dielectric rod is solved using the approximate semianalytical techniques of Marcatili, Knox, and Toulios. The accuracy with measured frequencies appears satisfactory with the former approach.**

I. Introduction

The dielectric resonators of cylindrical shape excited by the TE_{01} mode, are used in many miniaturised microwave circuits. The resonant frequency of this mode can be estimated quite accurately from various approximate models.

This paper deals with dielectric resonators of rectangular shape, for which only one attempt, that of Guillon and Garault

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